

Kaufman - Trading Systems & Methods - Chap04

Chap04

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Trend Calculations

The purpose of all trend identification methods is to see past the underlying noise in the market, those erratic moves that seem to be meaningless, and find the current direction of prices. Because there may be more than one trend at any one time, caused by shortterm events and longterm policy, it is possible to search for the strongest, or most dominant trend, or a minor trend that corresponds to your expected time frame. The technique that is used to uncover the right trend depends upon whether any of the trend characteristics are known. Does it have a seasonal or cyclic component. is it based on longterm monetary policy, or is it an overnight effect? The more you know about the reasons why prices trend, the better you will be able to find the most reliable calculation for separating the price direction from market noise.

Chapter 3, "Regression Analysis," produced forecasts by finding relationships between two or more price series. Once you know that there is a fundamental relationship between data, based on measuring the properties of dependence and correlation. a formula can be found that expresses one price movement in terms of the other prices and data. The predictive qualities of these methods are best when applied to data that has been seen before. that is, prices that are within the range of historic data. Forecasting reliability decreases sharply when values are based on extrapolation outside the previous occurrences. This phenomenon will also be true of other trending methods. Because of the way we test and define the final trend calculation, it is based on the movement of historical data: when prices move to new levels, the results of the model will often deteriorate.

FORECASTING AND FOLLOWING

There is a clear distinction between forecasting the trend and following the trend. Forecasting, predicting the future price, is much more difficult than following the trend. As shown in the previous chapter, it may involve combining several forecasts in a forecasted price with a confidence level. The further you look ahead, the lower the confidence.

The techniques most commonly used for evaluating price trends, prices both within prior ranges or at new levels are called trend following models. These forecasting models, they are only concerned with evaluating the trend. This analysis is normally used to conclude that prices are moving in an upward, or sideways direction. From this simple building blocks, traders develop rules of action and develop complex strategies of anticipation. These trending methods are more flexible than the traditional methods. To achieve success they introduce a lag. A great effort has been made to develop these trending methods.

In an autoregressive model, one or more of the previous prices are used to predict the next sequential price. If P_t represents today's price, P_{t+1} represents tomorrow's expected price will be

$$P_{t+1} = a_0 + a_1P_t + a_2P_{t-1} + \cdots + a_nP_1 + e$$

where each price is given a corresponding weighting a_i and combining them to give the expected price for tomorrow $P_{t+1} \pm e$ (where e represents an error factor). The use of yesterday's price alone to generate tomorrow's price:

$$P_{t+1} = a_0 + a_1 P_t + e$$

which you may also recognize as the formula for a straight line, $y = ax + b$.

The autoregressive model does not have to be linear; each price can be given a different weighting to give the best linear predictive quality. Thus each expected price P_{t+1} could be represented by a linear expression, $P_{t+1} = a_0 + a_1 P_t + a_2 P_{t-1} + e$, or by an exponential expression, $\ln P_{t+1} = a_0 + a_1 \ln P_t + a_2 \ln P_{t-1} + e$, which is commonly used in econometrics. These two expressions could then be combined to form an autoregressive form. In going from the simple to the complex, it is natural to want to know which choices will perform best. The answer can only be found by applying the model, validation, and experience. Various methods, based on a score, have been attempted and applied to actual data in real-time or extrapolated to the future to test the predictive quality of any model. Chapters 5 and 15 will discuss the methods that have been most popular and Chapter 21 will show testing methods that lead to robust results.

LEASTSQUARES MODEL

The leastsquares regression model is the same technique that was used in the previous chapter to find the relationship between two dependent variables, corn and soybeans, or to find how prices moved when driven by known related factors such as supply and demand. Here, the leastsquares model will be used to find the relationship between time and price, rather than between two prices, where the price forecast that we are seeking is dependent upon time. The regression model will also be applied in an autoregressive way by recalculating the expected price daily and using the slope of the resulting straight line or curvilinear fit to determine the direction of the trend.

A simple error analysis can be used to evaluate the predictive qualities of this method. Assume that there is a lengthy price series for a market and that we would like to know how many prior days are optimum for predicting the next day's price. The answer is found by looking at the average error in the predictions. If the number of days in the calculation increases and if the predictive error decreases, the answer is improving; if the error stops decreasing, the accuracy limit has been reached. Error analysis can improve most trend calculations, and it is also covered in the section on exponential smoothing later in this chapter. As an example, start by using only one prior day to find the price forecast

$$P_{t+1} = a_0 + a_1 P_t$$

$$P_t = a_0 + a_1 P_{t-1}$$

$$P_{t-1} = a_0 + a_1 P_{t-2}$$

⋮

$$P_2 = a_0 + a_1 P_1$$

and work up to a large number of days:

$$P_{t+1} = a_0 + a_1 P_t + a_2 P_{t-1} + \cdots + a_n P_{t-n+1}$$

$$P_t = a_0 + a_1 P_{t-1} + a_2 P_{t-2} + \cdots + a_n P_{t-n}$$

$$P_{t-1} = a_0 + a_1 P_{t-2} + a_2 P_{t-3} + \cdots + a_n P_{t-n-1}$$

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In the last case, it takes n days of prior prices to generate $n + 2$ equations can be written to solve the $n + 1$ coefficient matrix elimination method found in Appendix 3. The actual price P_t . We will use the notation $P'(n)_t$ to mean an n -day linear regression; therefore, $P'(3)_{25} = 58.00$ (day 25 was 58.00 using a 3-day linear regression and 3 days). The error occurring in each prediction is defined

$$E(n)_t = P - P'(n)_t$$

the difference between the actual and predicted values for that day using the n -day linear regression. As an example of error analysis, a copper sequence was selected for a period of 20 trading days that included a slight upward and a slight downward move, with some intermediate changes of direction. Table 41 shows the actual predictions using linear regressions for 2 through 7 prior prices and a prediction of one day forward. Table 42 shows the relative error in these predictions and an analysis of the errors. The mean, shown at the bottom of Table 42, is a simple average of all the points and gives the net trend bias during the sample period; the small range from negative to positive shows very little bias. The standard deviation measures the accuracy of the forecast. When the standard deviation is small, the predicted values are closer to the actual values and the forecast error is smaller.

The standard deviation and variance are both measurements of the distribution of points around the fitted line. As discussed in Chapter 2, the standard deviation measures the occurrence of points near the predictions; the smaller the value, the closer the grouping and the better the estimation. The variance is another way of looking at the deviations; the smaller the variance, the better the forecast. The copper error analysis shows the smallest values for $E(4)_n$ for the 4-day linear regression. This is an

interesting result because we would expect the shortest forecasting interval to produce the smallest error simply because prices cannot move as far from the forecast in 2 days as they can in 7 days. Therefore, we should expect the absolute size of the error, as measured by the standard deviation or variance, to get larger as the forecasting period gets larger. The smaller error produced by the 4day regression shows that there was a pattern during this short test that could be fit better by the 4day model than by any of the others.

Determination of the best predictive model using error analysis can be applied to any forecasting technique. This works particularly well when comparing the errors of two different forecasting methods evaluated over the same number of periods, eliminating the bias caused by longer and shorter intervals. It is also practical to carry the error analysis one step further and include the results of the prediction error on day $t+1, t+2$, and so on. This gives a measure of outofsample forecast accuracy and lends confidence to the predictive qualities of the technique

Having selected the most accurate forecast model, the size of the prior day predictive error can be used to resolve trading decisions. Consider the following situations:

1. The prediction and the actual price are very close (high confidence level). For example, the longterm copper error may have 1 standard deviation = .25.
2. Today's forecast error is within 1 standard deviation of expectations; therefore, we continue to follow the trend strategy.
3. Today's forecast error is between 1 and 3 standard deviations of expectations; therefore, we are cautious, yet understand that this is normal but less frequent.
4. Today's forecast error is greater than 3 standard deviations. This is unusual, indicates high risk, and may identify a price shock. Alternately, it could indicate a trend turning point.

TABLE 41 Analysis of Predictive Error for Copper

Date	Sequence (t)	Price P_t	Price Predictions for n-Day Linear Regression					
			$P'(2)_t$	$P'(3)_t$	$P'(4)_t$	$P'(5)_t$	$P'(6)_t$	$P'(7)_t$
10-21	1	60.30						
10-22	2	59.30						
10-25	3	58.70						
10-26	4	57.80						
10-27	5	59.80						
10-28	6	58.20						
10-29	7	58.40						
11-01	8	58.90	58.60	57.40	58.60	58.52	58.30	57.95
11-03	9	59.10	59.40	59.20	58.20	58.86	58.75	58.53
11-04	10	61.00	59.30	59.50	59.45	58.67	59.10	58.98
11-05	11	62.10	61.90	61.77	61.35	60.15	60.15	60.30
11-08	12	62.00	63.20	63.73	63.15	62.75	62.37	61.53
11-09	13	62.50	61.90	62.70	63.50	63.38	63.20	62.94
11-10	14	63.50	63.00	62.60	63.00	63.68	63.71	63.64
11-11	15	61.30	64.50	64.16	63.70	63.84	64.34	64.38
11-12	16	61.70	60.10	61.23	62.05	62.25	62.69	63.36
11-15	17	62.00	62.10	60.36	61.10	61.66	61.87	62.30
11-16	18	62.20	62.30	62.37	61.10	61.36	61.71	61.86
11-17	19	61.70	62.40	62.47	62.55	61.57	61.64	61.85
11-18	20	61.50	61.20	61.67	61.95	62.17	61.47	61.51
11-19	21	62.40	61.30	61.10	61.35	61.61	61.85	61.31
11-22	22	61.40	63.30	62.57	62.05	61.99	62.07	62.20
11-23	23	59.90	60.40	61.67	61.75	61.57	61.61	61.73
11-24	24	59.80	58.40	58.73	59.85	60.29	60.37	60.58
11-26	25	59.10	59.70	58.76	58.85	59.22	59.59	59.71
11-29	26	59.50	58.40	58.85	58.30	58.06	58.55	58.87
11-30	27	59.10	59.90	59.17	59.10	58.56	58.20	58.48

THE MOVING AVERAGE

The simplest and most wellknown of all smoothing techniques is called the moving average (MA). Using this method, the number of elements to be averaged remains the same,

but the time interval advances. Using a generalized

$P_2, \dots, P_t$ are a sequence of prices. A moving average of n data points, at time t would be

$$MA_t = \frac{P_t + P_{t-1} + \dots + P_{t-n+1}}{n} = \frac{\sum_{i=1}^n P_{t-i+1}}{n}, \quad n \leq t$$

In other words, the most recent moving average (mean) of the prior n data points. For example, u moving average:

$$\begin{aligned} \text{MA}_3 &= (P_1 + P_2 + P_3)/3 \\ \text{MA}_4 &= (P_2 + P_3 + P_4)/3 \\ &\vdots \\ \text{MA}_t &= (P_{t-2} + P_{t-1} + P_t)/3 \end{aligned}$$

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TABLE 42 Analysis of Predictive Error for Copper

Date	Sequence (t)	Price P_t	Predictive Error for an n-Day Linear Regression					
			$E(2)_t$	$E(3)_t$	$E(4)_t$	$E(5)_t$	$E(6)_t$	$E(7)_t$
10-21	1	60.30						
10-22	2	59.30						
10-25	3	58.70						
10-26	4	57.80						
10-27	5	59.80						
10-28	6	58.20						
10-29	7	58.40						
11-01	8	58.90	-.30	-1.50	-.30	-.38	-.60	-.95
11-03	9	59.10	.30	.10	-.90	-.24	-.35	-.57
11-04	10	61.00	-1.70	-1.50	-1.55	-2.33	-1.90	-2.02
11-05	11	62.10	-.20	-.33	-.75	-1.95	-1.95	-1.80
11-08	12	62.00	1.20	1.73	1.15	.75	.37	-.47
11-09	13	62.50	-.60	.20	1.00	.88	.70	.44
11-10	14	63.50	-.50	-.90	-.50	.18	.21	.14
11-11	15	61.30	3.20	2.86	2.40	2.54	3.04	3.08
11-12	16	61.70	-1.60	-.47	.35	.55	.99	1.66
11-15	17	62.00	.10	-1.64	-.90	-.34	-.13	.30
11-16	18	62.20	.10	.17	-1.10	-.84	-.49	-.34
11-17	19	61.70	.70	.77	.85	-.13	-.06	.15
11-18	20	61.50	-.30	.17	.45	.67	-.03	.01
11-19	21	62.40	-1.10	-1.30	-1.05	-.79	-.65	-1.09
11-22	22	61.40	1.90	1.17	.65	.59	.67	.80
11-23	23	59.90	.50	1.77	1.85	1.67	1.71	1.83
11-24	24	59.80	-1.40	-1.07	.05	.49	.57	.78
11-26	25	59.10	.60	-.34	-.25	.12	.49	.61
11-29	26	59.50	-1.10	-.70	-1.20	-1.44	-.95	-.63
11-30	27	59.10	.80	.07	.00	-.54	-.90	-.62
σ (Standard deviation)			1.209	1.217	1.069	1.151	1.149	1.220
M (Mean)			.030	-.037	.012	-.027	.037	.065
V (Variance)			1.389	1.406	1.086	1.260	1.255	1.415

If P_t represented a price at a specific time, the moving average would smooth the price movement. When more prices are used, the new price will be a smaller part of the average and have less effect on the final value. Five successive prices form a five day moving average. When the next sequential price is added and the oldest is dropped off, the prior average is changed by 1/3 of the difference between the old and the new values. If

$$MA_5 = (P_1 + P_2 + P_3 + P_4 + P_5)/5$$

and

$$MA_6 = (P_2 + P_3 + P_4 + P_5 + P_6)/5$$

then $c = P_2 + P_3 + P_4 + P_5$ can be substituted for the common terms in the moving average, and substituted to get

$$MA_6 = MA_5 + (P_t - P_{t-n})/5$$

This also gives a faster way to calculate a moving average. In terms in the moving average, the less effect the addition of a

$$MA_t = MA_{t-1} + (P_t - P_{t-n})/n$$

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The selection of the proper number of terms is based on both the technical consideration of the predictive quality of the choice (measured by the error) and the need to determine price trends over specific time periods for commercial use. The more days or data points used in the moving average, the more smoothing will occur; variation lasting only a short while will have less effect. There is also a danger of losing cyclic or seasonal price patterns by selecting the wrong value of n . For example, a repeating cycle of four data points 5, 8, 3, 6, 9, 4, 7, . . . , which advances by the value 1 each complete cycle, will appear as a straight line if a moving average of 4 days is used. If there is a possibility of a cyclic or seasonal pattern within the data, care should be taken to select a moving average that is out of phase with the possible pattern (that is, not equal to the cycle period).

The length of the moving average must also relate to its use. A jeweler may purchase silver each week to produce bracelets. Frequent purchases of small amounts keep the company's cash outlay small. The purchaser can wait as long as possible while prices continue to trend downward during any one week but will buy immediately when prices turn upward. A 6month trend cannot help his problem, because it gives a longterm answer to a shortterm issue; however, a 5day moving average may give the trend direction within the jeweler's time frame.

UserFriendly Software

Fortunately, we have reached a time when it is not necessary to perform these calculations the long way. Spreadsheet programs and specialized testing software provide simple tools for performing trend calculations as well as many other more complex functions discussed in this book. The notation for many of the different spreadsheets and software is very similar and selfexplanatory:

Function	Spreadsheet	
Sum	@sum(list, period)	@
Moving average	@avg(list, period)	@
Standard deviation	@std(list, period)	@

What Do You Average?

The closing or daily settlement is the most common price applied to a moving average. It is generally accepted as the true price of the day and is used by many analysts for calculation of trends. But other alternatives exist. The average of the high and low prices of the day will smooth the results by preventing the maximum difference from occurring when the close is also the high or low. Similarly, the closing price may be added to the high and low, and an average of the three used as the basis for the moving average.

Another valid component of a moving average can be other averages. For example, if

P_1 through P_n are prices, and MA_n is a 3-day moving average,

$$MA_3 = (P_1 + P_2 + P_3)/3$$

$$MA_4 = (P_2 + P_3 + P_4)/3$$

$$MA_5 = (P_3 + P_4 + P_5)/3$$

and

$$MA'_5 = (MA_3 + MA_4 + MA_5)/3$$

MA'_5 is a *double-smoothed* moving average, which

More on double smoothing can be found later

lowers independently is another technique that can

range, or volatility. This has been used to identify normal and extreme moves, and will also be discussed in Chapter 5 ("Trend Systems").

Types of Moving Averages

Besides varying the length of the moving average and the elements that are to be averaged, there are a great number of variations on the moving average.

An accumulative average may be used for a longterm trend. it does not satisfy our strict definition of a moving average, because it adds data but does not discard any; therefore, it is cumulative. It is traditionally started at the beginning of a futures contract and continued until the contract expires. It may be applied to stocks after each dividend notice. Due to the accumulation of prices and the constant increase in the length of the average, the effect of the additional price at day t on the moving

average will be $(P_t - \text{AVG})/t$, where

AVG is the average of all prices from the beginning through t . The impact of 1 day becomes very small toward the end of a 1year contract, which has approximately 250 trading days. A reset accumulative average is a modification of the standard accumulative average and attempts to correct for the loss of sensitivity as the number of trading days becomes large. This alternative allows you to reset or restart the moving average whenever a new trend begins, a significant event occurs, or at some specified time interval.

Truncated moving averages are the most common, and they are often called simple moving averages. The most basic has already been discussed in detail. An alternate method of calculating a daily moving average calculation is to keep the total of the past n days. Each new day then only requires the addition of the new value and subtraction of the oldest one. The new total is saved for the next day and is also divided by n to get the new moving average value. An interesting twist to this technique is the averagemodified method, in which the price of the new day is added and the last moving average value is subtracted. Returning to the example of a 5day moving average, day 6 was

$$\mathbf{MA_6 = MA_5 + (P_6 - P_1)/5}$$

This becomes

$$\mathbf{MA_6 = MA_5 + (P_6 - MA_5)/5}$$

The averagemodified version is convenient, because only the prior moving average value and the new price are necessary for each calculation. The substitution of the moving average value tends to smooth the results even more than a simple moving average. Its use

prevents the difference $(P_6 - P_1)$ from becoming too extreme; it effectively cuts the possible range in half and dampens the endoff impact.

The weighted moving average opens many possibilities. It allows the significance of individual or groups of data to be changed. it may restore proper value to parts of a data sample, or it may incorrectly bias the data. A weighted moving average is expressed in its general form as

$$\mathbf{W_t = \frac{w_1P_t + w_2P_{t-1} + \cdots + w_nP_{t-n}}{w_1 + w_2 + \cdots + w_n}}$$

This gives the weighted moving average at time t as the average of the previous n prices, each with its own weighting factor w . The most popular form of this technique is called front loaded, because it gives more weight to the most recent data and reduces the significance of the older elements. Therefore, for the frontloaded weighted moving average (see Figure 41)

$$w_1 \geq w_2 \geq \cdots \geq w_n$$

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The weighting factors w_i may also be determined by regression analysis, but then they may not necessarily be frontloaded. A common modification to front loading is called step

weighting in which each successive w_i differs from the previous by a fixed increment

$$c = w_i - w_{i-1}$$

The simplest case takes integer values for c

$$w_1 = n$$

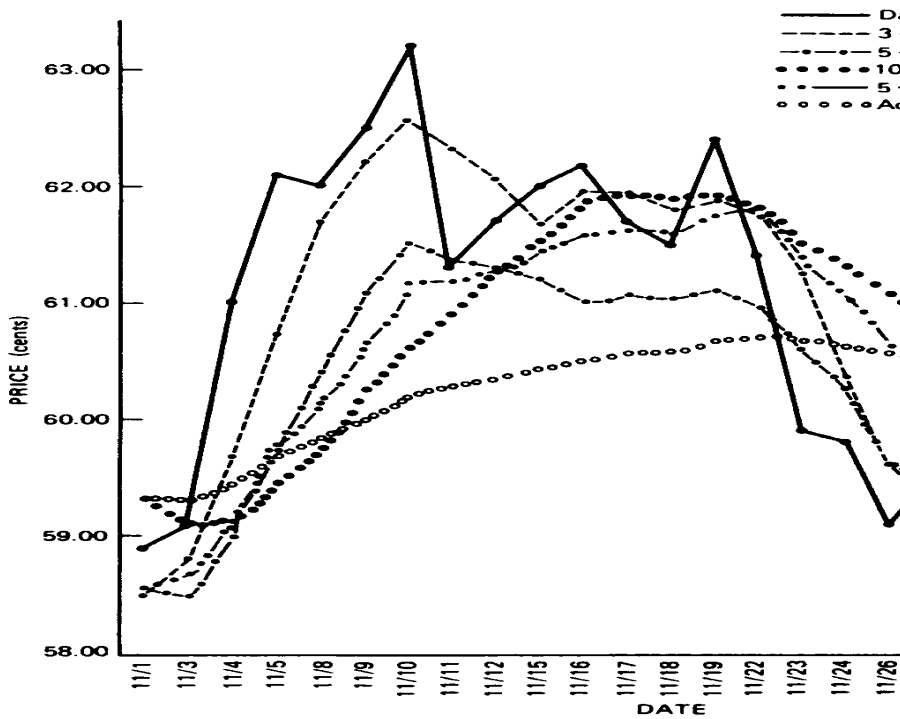
$$w_2 = n - 1$$

$\vdots$

$$w_n = 1$$

This gives the weighting factors the values of 5, 4, 3, 2, and 1 for a 5day average. A computer program for calculating a linearly weighted moving average is

FIGURE 41 A comparison of moving averages.



```

1      sum = 0;
2      for I = 1 to N begin
3          M = N - I + 1;
4          sum = sum + M*price;
5      end;
6      LWMA = sum/ ( N* (N+1) /2 )

```

In line 6, the divisor $(N*(N+1))/2$ is the arithmetic sum of the integers from 1 to N. Another approach would be a percentage relationship

$$w_{i-1} = a \times w_i$$

where $a = .90$; then, $w_5 = 5$, $w_4 = 4.5$, $w_3 = 4.05$, etc.

Prices may also be weighted in groups. If each group is given the same weighting factor,

$$W_t = \frac{w_1 P_t + w_1 P_{t-1} + w_2 P_{t-2} + w_3 P_{t-3} + \dots + w_{n/2} P_{t-n/2+1}}{2 \times (w_1 + w_2 + \dots + w_{n/2})}$$

or, grouped with n even,

$$W_t = \frac{w_1 (P_t + P_{t-1}) + w_2 (P_{t-2} + P_{t-3}) + \dots + w_{n/2} (P_{t-n/2+1} + P_{t-n/2})}{2 \times (w_1 + w_2 + \dots + w_{n/2})}$$

Any number of consecutive data elements can be grouped for a stepweighted moving average. Because there can be any number of combinations, there is no simple, automatic method for calculating anything other than a linearly weighted average, which is the function `WAverage(price, length)` in Omega's Easy Language. If the purpose of weighting is to duplicate a pattern that is intrinsic to price movement, then the following sections on geometric averages and exponential smoothing may provide better tools.

These moving averages can also be plotted in different ways, each way having a major impact on their interpretation. The conventional plot places the moving average value MA, on the same vertical line as the last entry P_t of the moving average. When prices have been trending higher over the period of calculation, this will cause the value MA to lag behind (or below) the actual prices., when prices are declining, the moving average will be above the prices (see Figure 42).

The plotted moving average can either lead or lag the last price recorded. if it is to lead by 3 days, the value MA, is plotted on the vertical line $t + 3$; if it is to lag by 2 days, it is plotted at $t - 2$. In the case of leading moving averages, the analysis attempts to compensate for the time delay by judging price direction using a forecast based on current rate of change and direction. A penetration of the forecasted line by the price may be used to signal a change of direction. The lag technique may also serve the more

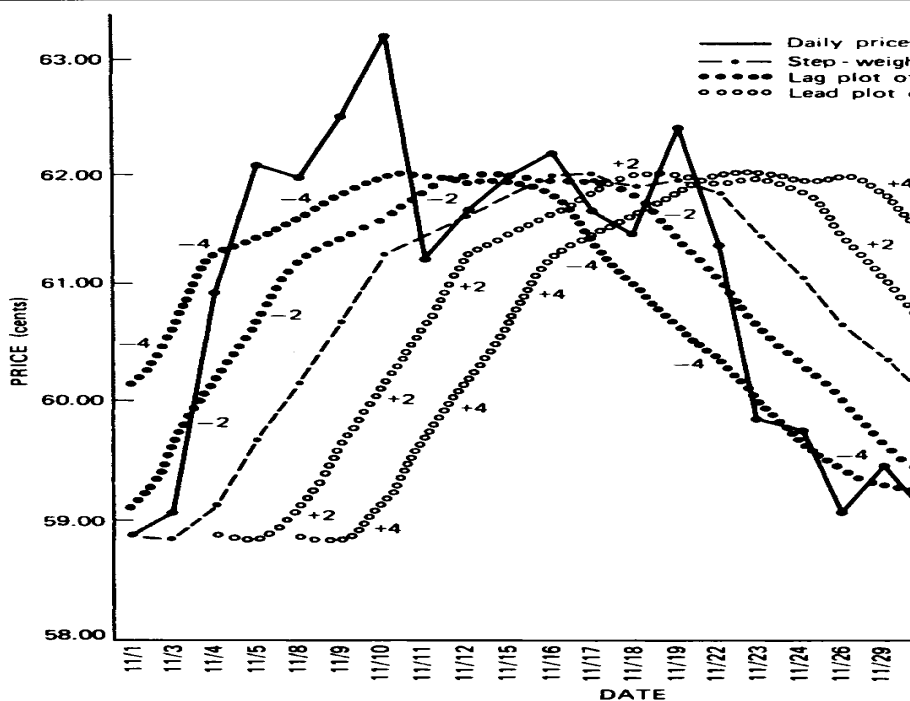
sophisticated purpose of smoothing the moving average. A 10day moving average, when lagged by 5 days, will be plotted in the middle of the actual price data. This technique will be covered later.

Comparison of Moving Average Methods

The comparison seen in Table 43 confirms that simple moving averages taken over shorter periods identify shorter trends. The 3day moving average trendline had 9 separate trends, while the 10day had only 5. The 5day average modified method was very similar to the 10day simple moving average due to removing the average price each day. The cumulative average showed the fewest trends even though it began its accumulation process only 10 days prior; as time increases, this trend will become less responsive. For this short sample interval, the 10day linearly weighted average was identical to the average modified method and very similar to the 10day simple average; however, because it weights the recent data 10 times greater than the oldest, this similarity will not continue. A closer look will show that the trend values are very different.

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FIGURE 4.2 Plotting lag and lead for a 10day stepweighted moving average with weighting factors of 10, 9, 8, . . .



Triangular Weighting

While the simple moving average or linear regression treats each price equally, exponential smoothing and linear step weighting puts greater weight on the most recent data. There is an entire area of study in which the period of the dominant cycle is the basis for determining the best trend

period. Triangular filtering' is a related concept that attempts to uncover the trend by reducing the noise on both the front and rear of the trend window, where it is expected to have the greatest interference. Therefore, if a 20day weighted average is used, the 10th day will have the greatest weight, while days 1 and 20 will have the smallest.

To implement triangular weighting, begin with the standard formula for a weighted average,

$$W_t = \frac{\sum_{t=1}^n w_t P_t}{\sum_{t=1}^n w_t}$$

I jj. Payne, 'A better way to smooth data," Technical Analysis of Stocks & Commodities (October 1989).

TABLE 43 Comparison of Moving Averages Methods*

	Price	Simple Moving Averages			5-Day Average Modified	Cum Average	10-Day Linearly Weighted
		3-Day	5-Day	10-Day			
930316	64.59						
930317	64.23						
930318	65.26						
930319	65.24						
930322	65.07						
930323	65.14						
930324	64.98						
930325	64.76						
930326	65.11						
930329	65.46	65.11	65.09	64.98	78.10	65.46	65.08
930330	65.94	65.50	65.25	65.12	75.67	65.07	65.26
930331	66.10	65.83	65.47	65.31	73.76	65.16	65.43
930401	66.87	66.30	65.90	65.47	72.38	65.29	65.72
930402	66.56	66.51	66.19	65.60	71.22	65.38	65.92
930405	66.71	66.71	66.44	65.76	70.31	65.47	66.12
930406	66.19	66.49	66.49	65.87	69.49	65.51	66.20
930407	66.14	66.35	66.49	65.98	68.82	65.55	66.25
930408	66.64	66.32	66.45	66.17	68.38	65.61	66.37
930412	67.33	66.70	66.60	66.39	68.17	65.70	66.58
930413	68.18	67.38	66.90	66.67	68.17	65.83	66.90
930414	67.48	67.66	67.15	66.82	68.04	65.90	67.05
930415	67.19	67.62	67.36	66.93	67.87	65.96	67.12
930416	66.46	67.04	67.33	66.89	67.59	65.98	67.03
930419	67.20	66.95	67.30	66.95	67.51	66.03	67.09
930420	67.62	67.09	67.19	67.04	67.53	66.10	67.21
930421	67.66	67.49	67.23	67.19	67.56	66.16	67.32
930422	67.89	67.72	67.37	67.37	67.62	66.22	67.45
930423	69.19	68.25	67.91	67.62	67.94	66.33	67.78
930426	69.68	68.92	68.41	67.86	68.29	66.44	68.15
930427	69.31	69.39	68.75	67.97	68.49	66.54	68.42
930428	69.11	69.37	69.04	68.13	68.61	66.62	68.63
930429	69.27	69.23	69.31	68.34	68.75	66.71	68.83
930430	68.97	69.12	69.27	68.59	68.79	66.77	68.95
930503	69.11	69.12	69.15	68.78	68.85	66.84	69.04
930504	69.50	69.19	69.19	68.97	68.98	66.92	69.17
930505	69.70	69.44	69.31	69.17	69.13	67.00	69.31
930506	69.94	69.71	69.44	69.38	69.29	67.08	69.45
930507	69.11	69.58	69.47	69.37	69.25	67.13	69.40
930510	67.64	68.90	69.18	69.17	68.93	67.14	69.08
930511	67.75	68.17	68.83	69.01	68.69	67.16	68.83
930512	67.47	67.62	68.38	68.85	68.45	67.16	68.55
930513	67.50	67.57	67.89	68.67	68.26	67.17	68.30
930514	68.18	67.72	67.71	68.59	68.24	67.20	68.21
930517	67.35	67.68	67.65	68.41	68.07	67.20	67.99
930518	66.74	67.42	67.45	68.14	67.80	67.19	67.68
930519	67.00	67.03	67.35	67.87	67.64	67.19	67.48
930520	67.46	67.07	67.35	67.62	67.60	67.19	67.40
930521	67.36	67.27	67.18	67.45	67.56	67.19	67.35
930524	67.37	67.40	67.19	67.42	67.52	67.20	67.34
930525	67.78	67.50	67.39	67.42	67.57	67.21	67.41
930526	67.96	67.70	67.59	67.47	67.65	67.22	67.50
No. Trends		11	9	5	5	3	5

* The gray areas of the table show downtrends.

where n is the size, or width, of the window. The weights increase from 1 to the middle of the window, at $n/2$, then decrease in the same different form when the period is odd or even,

$$\begin{aligned} w_i &= i, & \text{for } i &= 1 \text{ to } \text{@intportion}((n+2)/2) - 1 \\ & n - i + 1, & \text{for } i &= \text{@intportion}((n+2)/2) \\ & n - i, & \text{for } i &= \text{@intportion}((n+2)/2) + 1 \end{aligned}$$

where, for odd values of n , the weight factor has the value $n/2$ (rounded up using the function for the integer portion of a number). For even n . Instead of a triangular filter, which is very precise for the greatest importance, we may choose a Gaussian filter, which is more similar to a bell curve. Here, the weighting factors are

$$w_i = 10^x \quad \text{and} \quad x = \frac{3}{2} \left(1 - \frac{2i}{n} \right)^2$$

PivotPoint Weighting

Too often we limit ourselves by our training. When a weighted moving average is used, it is normal to assume that all the weighting factors should be positive; however, that is not a requirement. The pivotpoint moving average uses linear weights (e.g., 5, 4, ...) that begin with a positive value and continue to decline even when they become negative.' This method treats the oldest data in a manner opposite to their direction. In the following formula, a pivot point, where the weighting is zero, is reached about 2/3 of the way through the data interval. For a pivotpoint moving average of 11 values, the 8th data point is weighted with 0:

$$\text{PPMA}_t(11) = (7P_t + 6P_t + 5P_t + 4P_t + 3P_t + 2P_t + 1P_t + 0P_t + 1P_t + 2P_t + 3P_t + 4P_t + 5P_t + 6P_t + 7P_t) / 11$$

This technique is intended to limit the lag by front loading the prices and reducing the divisor of the weighting formula by the sum of the negative weighting factors. The general formula for an nday pivotpoint moving average W

$$\text{PPMA}_t(n) = \frac{2}{n(n+1)} \left(\sum_{i=1}^{n/2} i P_{t-i} + \sum_{i=1}^{n/2} i P_{t+i} \right)$$

To implement this in a computer program, the following steps would be used:

```

1      sum = 0;
2      for I = 1 to N be
3          M = N - I + 1
4          sum = sum + P
5      end;
6      PPMA = sum*(2/ (N

```

where lines 2 through 5 calculate the sum of the prices times the weighting factor, and line 6 divides that sum by the sum of the weighting factors (after multiplying the top and bottom of the fraction by the value 2).

The negative weighting factors reverse the impact of the price move for the oldest data points rather than just give them less importance. For a short interval this can cause the trendline to be outofphase with prices. The application of this method should be limited to longerterm cyclic markets, where the reflection point, at which the weighting factor becomes zero, aligns with the cyclic rum or can be fixed at the point of the last trend change.

¹Patrick E. Lafferry, "EndPoint Moving Average," TecAmical Analysis of Stocks & Commodities (October 1995).

¹ Don Kraska, "Letters To S&C," TecAmical Analysis of Stocks & Commodities (February 1996, p. 12).

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GEOMETRIC MOVING AVERAGES

The geometric mean is a growth function that is very applicable to longterm price movement, introduced in Chapter 2 ("Basic Concepts"). It is especially useful calculating the components of an index. The geometric mean can also be applied to the most recent n points at time t to get a geometric average similar to a moving average

$$G_t = (P_t \times P_{t-1} \times \cdots \times P_{t-n})^{1/n}$$

The daily calculation is more complicated but, as shown in Chapter 2, could be rewritten as

$$\ln G_t = \frac{\ln P_t + \ln P_{t-1} + \cdots + \ln P_{t-n+1}}{n}$$

$$= \frac{1}{n} \left(\sum_{i=1}^n \ln P_{t-i+1} \right)$$

This is similar in form to the summation of a standard moving average based on the arithmetic mean and can be written for either spreadsheet or program code as

`GA = @average(@log(price),n)`

Note that some software will use “log” although “ln” is more correct. Other programs will allow the choice of @log. The geometric moving average would have the form

$$\ln G_t = \frac{w_1 \ln P_t + w_2 \ln P_{t-1} + \cdots + w_n \ln P_{t-n+1}}{w_1 + w_2 + \cdots + w_n}$$

$$= \frac{\sum_{i=1}^n w_i \ln P_{t-i+1}}{\sum_{i=1}^n w_i}$$

The geometric moving average itself would give greater weight to smaller values without the need for a discrete weighting function. In applying the technique to actual commodity prices, this distinction may not be obvious. For widely ranging values such as 1,000 and 10, the simple average is 505 and the geometric average is 100, but for the three sequential cocoa prices, 56.20, 58.30, and 57.15, the arithmetic mean is 57.2166 and the geometric is 57.1871. A similar test of 5, 10, or 20 days of commodity prices will show a negligible difference between the results of the two averages. If the geometric moving average is to be helpful, it would be best applied to longterm historic data with wide variance, using yearly or quarterly average prices.

DROPOFF EFFECT

Two types of trending methods can be distinguished by the dropoff effect, a common way of expressing the abrupt change in value when a fixedperiod calculation drops off a significant value. A simple moving average, linear regression, and weighted averages all use a fixed period, or window, for determining the trend. When an unusually volatile piece of data becomes old and drops off, there can be a sudden jump in the

trend value, called the dropoff effect. For an nperiod moving average, this value is the difference between the first and the last items, divided by the number of periods,

$$\text{drop-off effect} = (P_1 - P_n) / n$$

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A weighted average, in which the oldest values have less importance, reduces this effect because the highvolatility data slowly become a smaller part of the result before being dropped off. Exponential smoothing, discussed next, is immune from this problem by its very nature.

EXPONENTIAL SMOOTHING

Exponential smoothing may appear to be more complex than other techniques, but it is only another form of a weighted moving average. It has the added advantage of being simpler to calculate than any other method discussed; only the last exponentially smoothed

value E_{t-1} and the smoothing constant a are necessary. The technique of exponential smoothing was developed by Brown in 1963. It is a method of projecting their position—the immediate past is used.

The geometric progression

$$1, a, a^2, a^3, \dots, a^{n-1}$$

applied to the terms of a weighted moving average

$$W_t = \frac{w_1 P_t + w_2 P_{t-1} + \dots + w_n P_{t-n+1}}{w_1 + w_2 + \dots + w_n}$$

gives $w_1 = 1, w_2 = a, w_3 = a^2, \dots, w_n = a^{n-1}$. If $a = 0.5$, the resulting sequence is

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, (\frac{1}{2})^{n-1}$$

This shows the rapidly decreasing importance of older data. The geometric progression into the equation for the weighted moving average

$$E_t = \frac{1P_t + aP_{t-1} + a^2P_{t-2} + \dots + a^{n-1}P_{t-n+1}}{1 + a + a^2 + \dots + a^{n-1}}$$

By a lengthy arithmetic process using the formula for the sum of a geometric progression, the same equation can be stated as

$$E_t = (1 - a)P_t + aE_{t-1}$$

where P_t is the most recent price and $0 \leq a \leq 1$. If $a = 0.3$, the value of past prices is distributed such that $a \times 100\%$ of the total moving average and the balance to the most recent price. If $a = 0.3$, the price receive a weighting of 30% of the total moving average. If $a = 0.1$, it reverses the weighting notations and has one less weight than the most recent price.

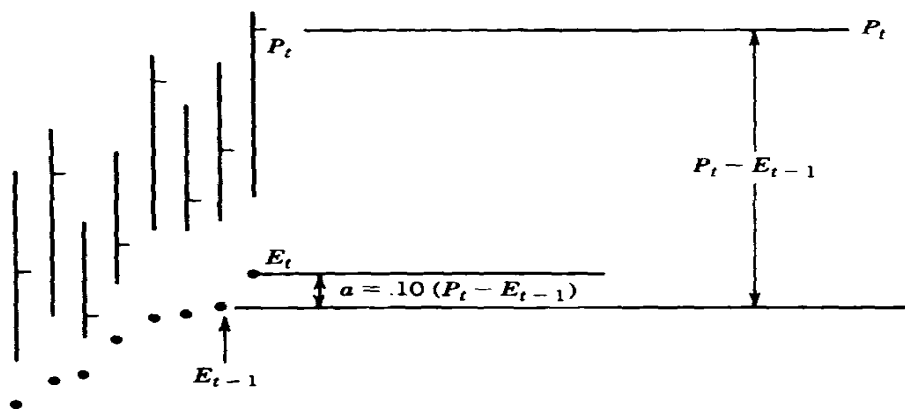
$$E_t = E_{t-1} + a (P_t - E_{t-1})$$

The smoothing process can be started by letting $E_1 = P_1$.

$$E_2 = E_1 + a (P_2 - E_1)$$

The smoothing constant, a , will determine how quickly the data will wind up the data. A longer-term smoothing, where a is small, will take longer for the smoothed line, E_t , to reach a stable value. This can be seen in Figure 4-3 as

New exponential value = prior exponential value + some % of current price minus prior exponential value



An important feature of the exponentially smoothed moving average is that all data previously used are always part of the new result, although with diminishing significance. in general:

$$E_t = a (P_t + (1 - a)P_{t-1} + (1 - a)^2P_{t-2} + \cdots + (1 - a)^{t-1}P_1)$$

For example, if the smoothing constant $a = .10$, the new smoothed average is 10% of the current data point plus 90% of the old average:

$$E_t = E_{t-1} + .10 (P_t - E_{t-1})$$

Each new calculation reduces all data from previous calculations. A calculation for $t + 1$ will cause the data from t through 1 to be reduced. Therefore, at any time t the impact of data used at a previous time $t - k$, k days elapsed, $t - k$, and the smoothing constant a determine the significance of that data. Then on day k we have

$$k = a \cdot D_k$$

$$k + 1 = a \cdot D_k - a \cdot a \cdot D_k$$

⋮

$$k + n = a \cdot D_k - (a^2 \cdot D_k + a^3 \cdot D_k + \cdots + a^{n+1} \cdot D_k)$$

$$= a \cdot D_k - \sum_{i=2}^{n+1} a^i D_k \quad (\text{written in summation form})$$

$$= d_k(a - \sum_{i=2}^{n+1} a^i) \rightarrow 0 \quad \text{as} \quad n \rightarrow \infty$$

This shows that the significance of the data on day k approaches zero as n approaches infinity. Consider the following example. An investor makes a purchase of shares of stock in a corporation in which there are t investors. The investor takes 10% of the shares. Then there are now $t + 1$ investors, and the original t shares are now owned by $t + 1$ investors. There are now $t + 1$ investors, and the original t shares are now owned by $t + 1$ investors. There are now $t + 2$ investors, and the investor t has now only 9% of the shares. The significance of the data on day k is now 9% of the original 10%.

investors are added, the stock holding dwindles to 7.29%, 6.561%, and 5.9049%. Even though the number of shares that are held remain the same, each previous owner holds a less significant part of the whole. No matter how many investors are added at 10%, the original shareholders will always have some small percent of the whole. In exactly the same way, the original price used in an exponentially smoothed moving average always retains some relevance., with a standard moving average of n days, the $(n + 1)$ th day is dropped off and ceases to have any impact.

Smoothing and Restoring the Lag

As a trend continues in its direction, the exponentially smoothed moving average will lag farther behind. By selecting a smoothing constant nearer to 1, the magnitude of this lag will be lessened but it will still increase, just as using a smaller number of days, or shorter period, will limit the lag of a simple moving average. If the lag is considered the predictive error e_t , in the exponential smoothing calculation, then

$$e_t = P_t - E_t$$

where E_t is the exponential smoothing approximation, the exponential smoothing technique can be applied to the predictive error to get

$$ERR_t = ERR_{t-1} + a(e_t - ERR_{t-1})$$

and then add the difference between the original smoothing value and this secondorder smoothing back into the approximation:

$$EE_t = E_t + ERR_t$$

By measuring the new error, the difference between the firstorder exponential E , and the secondorder EE , it can be seen whether there has been an improvement in the forecast. This process can be continued to thirdorder smoothing by applying the same method to the difference between the current price and the secondorder trend EE . A comparison of this method, restoring the forecast error, is compared with other exponential smoothing techniques in **Figure 44** and Table 44.

Double Smoothing

To make a trendline smoother, the period of a moving average or the exponential smoothing constant is normally increased. This succeeds in reducing the shortterm market noise at the cost of increasing the lag. It has been suggested that each of these trending methods could be double-smoothed, that is, the trend values can themselves be smoothed. This will slow down the trendline, but gives weight to the previous values in a way that may be unexpected.

A double-smoothed simple 3day moving average, MA, would take the original three moving average values and average them to get a double-smoothed moving average, DMA:

$$MA_3 = (P_1 + P_2 + P_3)/3$$

$$MA_4 = (P_2 + P_3 + P_4)/3$$

$$MA_5 = (P_3 + P_4 + P_5)/3$$

then

$$DMA_5 = (MA_3 + MA_4 + MA_5)/3$$

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TABLE 4-4 Comparison of exponential smoothing techniques applied to Swiss franc from TradeStation continuous daily data

	Price	Exp	Dbl Exp	Exp Err	Exp Error	Smoothed Error	Price Change	Exp Change	Dbl Exp Change
930316	64.59	64.59	64.59	64.59	0.00	0			
930317	64.23	64.55	64.59	64.52	-0.32	-0.03	-0.36	-0.04	-0.04
930318	65.26	64.62	64.59	64.66	0.64	0.03	1.03	0.07	-0.03
930319	65.24	64.69	64.60	64.77	0.55	0.09	-0.02	0.06	-0.02
930322	65.07	64.72	64.61	64.84	0.35	0.11	-0.17	0.04	-0.01
930323	65.14	64.77	64.63	64.90	0.37	0.14	0.07	0.04	-0.01
930324	64.98	64.79	64.64	64.93	0.19	0.14	-0.16	0.02	-0.01
930325	64.76	64.78	64.66	64.91	-0.02	0.13	-0.22	-0.00	-0.01
930326	65.11	64.82	64.67	64.96	0.29	0.14	0.35	0.03	-0.00
930329	65.46	64.88	64.69	65.07	0.58	0.19	0.35	0.06	0.01
930330	65.94	64.99	64.72	65.25	0.95	0.26	0.48	0.11	0.02
930331	66.10	65.10	64.76	65.44	1.00	0.34	0.16	0.11	0.02
930401	66.87	65.28	64.81	65.74	1.59	0.46	0.77	0.18	0.04
930402	66.56	65.40	64.87	65.94	1.16	0.53	-0.31	0.13	0.05
930405	66.71	65.53	64.94	66.13	1.18	0.60	0.15	0.13	0.06
930406	66.19	65.60	65.00	66.20	0.59	0.60	-0.52	0.07	0.06
930407	66.14	65.65	65.07	66.24	0.49	0.58	-0.05	0.05	0.06
930408	66.64	65.75	65.14	66.37	0.89	0.62	0.50	0.10	0.06
930412	67.33	65.91	65.21	66.61	1.42	0.70	0.69	0.16	0.07
930413	68.18	66.14	65.31	66.97	2.04	0.83	0.85	0.23	0.09
930414	67.48	66.27	65.40	67.14	1.21	0.87	-0.70	0.13	0.09
930415	67.19	66.36	65.50	67.23	0.83	0.86	-0.29	0.09	0.09
930416	66.46	66.37	65.59	67.16	0.09	0.79	-0.73	0.01	0.08
930419	67.20	66.46	65.67	67.24	0.74	0.78	0.74	0.08	0.08
930420	67.62	66.57	65.76	67.38	1.05	0.81	0.42	0.12	0.09
930421	67.66	66.68	65.86	67.51	0.98	0.83	0.04	0.11	0.09
930422	67.89	66.80	65.95	67.65	1.09	0.85	0.23	0.12	0.09
930423	69.19	67.04	66.06	68.02	2.15	0.98	1.30	0.24	0.11
930426	69.68	67.30	66.18	68.43	2.38	1.12	0.49	0.26	0.12
930427	69.31	67.51	66.32	68.69	1.80	1.19	-0.37	0.20	0.13
930428	69.11	67.67	66.45	68.88	1.44	1.21	-0.20	0.16	0.13
930429	69.27	67.83	66.59	69.06	1.44	1.24	0.16	0.16	0.14
930430	68.97	67.94	66.72	69.16	1.03	1.22	-0.30	0.11	0.13
930503	69.11	68.06	66.86	69.26	1.05	1.20	0.14	0.12	0.13
930504	69.50	68.20	66.99	69.41	1.30	1.21	0.39	0.14	0.13
930505	69.70	68.35	67.13	69.58	1.35	1.22	0.20	0.15	0.13
930506	69.94	68.51	67.27	69.76	1.43	1.24	0.24	0.16	0.14
930507	69.11	68.57	67.40	69.74	0.54	1.17	-0.83	0.06	0.13
930510	67.64	68.48	67.50	69.45	-0.84	0.97	-1.47	-0.09	0.11
930511	67.75	68.40	67.59	69.21	-0.65	0.81	0.11	-0.07	0.09
930512	67.47	68.31	67.67	68.96	-0.84	0.65	-0.28	-0.09	0.07
930513	67.50	68.23	67.72	68.74	-0.73	0.51	0.03	-0.08	0.06
930514	68.18	68.23	67.77	68.68	-0.05	0.45	0.68	-0.01	0.05
930517	67.35	68.14	67.81	68.47	-0.79	0.33	-0.83	-0.09	0.04
930518	66.74	68.00	67.83	68.17	-1.26	0.17	-0.61	-0.14	0.02
930519	67.00	67.90	67.84	67.96	-0.90	0.06	0.26	-0.10	0.01
930520	67.46	67.85	67.84	67.87	-0.39	0.02	0.46	-0.04	0.01

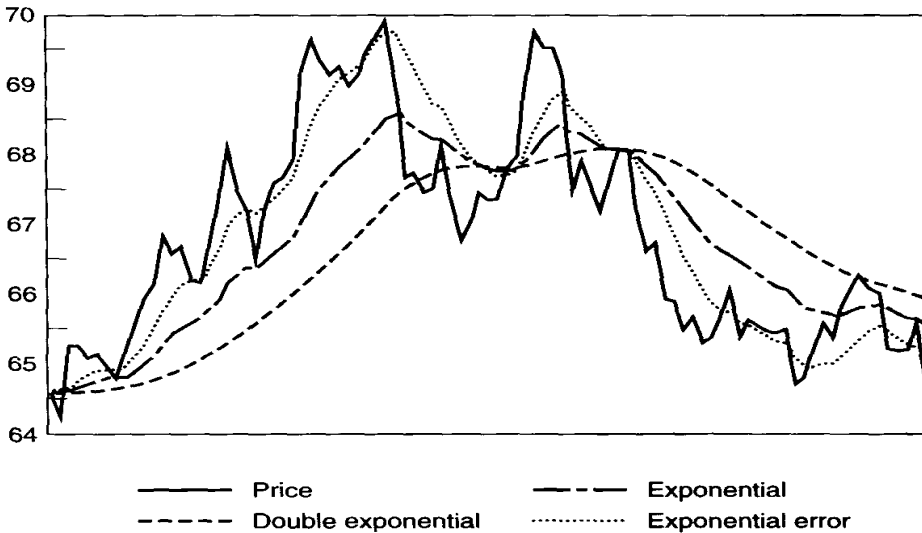
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TABLE 4-4 (Continued)

	Price	Exp	Dbl Exp	Exp Err	Exp Error	Smoothed Error	Price Change	Exp Change	Dbl Exp Change
930521	67.36	67.80	67.83	67.78	-0.44	-0.03	-0.10	-0.05	-0.00
930524	67.37	67.76	67.83	67.70	-0.39	-0.07	0.01	-0.04	-0.01
930525	67.78	67.76	67.82	67.71	0.02	-0.06	0.41	0.00	-0.01
930526	67.96	67.78	67.82	67.75	0.18	-0.03	0.18	0.02	-0.00
930527	68.97	67.90	67.82	67.98	1.07	0.08	1.01	0.12	0.01
930528	69.79	68.09	67.85	68.33	1.70	0.24	0.82	0.19	0.03
930601	69.53	68.23	67.89	68.58	1.30	0.34	-0.26	0.14	0.04
930602	69.52	68.36	67.94	68.79	1.16	0.43	-0.01	0.13	0.05
930603	69.17	68.44	67.99	68.90	0.73	0.46	-0.35	0.08	0.05
930604	67.41	68.34	68.02	68.66	-0.93	0.32	-1.76	-0.10	0.04
930607	67.96	68.30	68.05	68.55	-0.34	0.25	0.55	-0.04	0.03
930608	67.62	68.23	68.07	68.40	-0.61	0.16	-0.34	-0.07	0.02
930609	67.18	68.13	68.08	68.18	-0.95	0.05	-0.44	-0.11	0.01
930610	67.65	68.08	68.08	68.09	-0.43	0.01	0.47	-0.05	0.00
930611	68.08	68.08	68.08	68.09	-0.00	0.00	0.43	-0.00	0.00
930614	68.06	68.08	68.08	68.08	-0.02	0.00	-0.02	-0.00	0.00
930615	67.16	67.99	68.07	67.91	-0.83	-0.08	-0.90	-0.09	-0.01
930616	66.59	67.85	68.05	67.65	-1.26	-0.20	-0.57	-0.14	-0.02
930617	66.77	67.74	68.01	67.46	-0.97	-0.28	0.18	-0.11	-0.03
930618	65.95	67.56	67.97	67.15	-1.61	-0.41	-0.82	-0.18	-0.05
930621	65.89	67.39	67.91	66.87	-1.50	-0.52	-0.06	-0.17	-0.06
930622	65.48	67.20	67.84	66.56	-1.72	-0.64	-0.41	-0.19	-0.07
930623	65.69	67.05	67.76	66.34	-1.36	-0.71	0.21	-0.15	-0.08
930624	65.30	66.88	67.67	66.08	-1.58	-0.80	-0.39	-0.18	-0.09
930625	65.37	66.73	67.58	65.87	-1.36	-0.85	0.07	-0.15	-0.09
930628	65.70	66.62	67.48	65.76	-0.92	-0.86	0.33	-0.10	-0.10
930629	66.12	66.57	67.39	65.75	-0.45	-0.82	0.42	-0.05	-0.09
930630	65.37	66.45	67.30	65.61	-1.08	-0.85	-0.75	-0.12	-0.09
930701	65.64	66.37	67.21	65.54	-0.73	-0.83	0.27	-0.08	-0.09
930702	65.54	66.29	67.11	65.46	-0.75	-0.83	-0.10	-0.08	-0.09
930706	65.47	66.21	67.02	65.39	-0.74	-0.82	-0.07	-0.08	-0.09
930707	65.45	66.13	66.93	65.33	-0.68	-0.80	-0.02	-0.08	-0.09
930708	65.50	66.07	66.85	65.29	-0.57	-0.78	0.05	-0.06	-0.09
930709	64.70	65.93	66.76	65.11	-1.23	-0.82	-0.80	-0.14	-0.09
930712	64.82	65.82	66.66	64.98	-1.00	-0.84	0.12	-0.11	-0.09
930713	65.27	65.76	66.57	64.96	-0.49	-0.81	0.45	-0.05	-0.09
930714	65.62	65.75	66.49	65.01	-0.13	-0.74	0.35	-0.01	-0.08
930715	65.36	65.71	66.41	65.01	-0.35	-0.70	-0.26	-0.04	-0.08
930716	65.85	65.73	66.34	65.11	0.12	-0.62	0.49	0.01	-0.07
930719	66.09	65.76	66.29	65.24	0.33	-0.52	0.24	0.04	-0.06
930720	66.28	65.81	66.24	65.39	0.47	-0.42	0.19	0.05	-0.05
930721	66.07	65.84	66.20	65.48	0.23	-0.36	-0.21	0.03	-0.04
930722	66.03	65.86	66.16	65.55	0.17	-0.31	-0.04	0.02	-0.03
930723	65.22	65.79	66.13	65.46	-0.57	-0.33	-0.81	-0.06	-0.04
930726	65.19	65.73	66.09	65.38	-0.54	-0.35	-0.03	-0.06	-0.04
930727	65.19	65.68	66.05	65.31	-0.49	-0.37	0.00	-0.05	-0.04
930728	65.59	65.67	66.01	65.33	-0.08	-0.34	0.40	-0.01	-0.04
930729	64.80	65.58	65.97	65.20	-0.78	-0.38	-0.79	-0.09	-0.04

FIGURE 4-4 Comparison of exponential smoothing technique (Exp) is compared with a double-smoothed series (Dbl Exp) constant .10 first to prices, then to the smoothed series, and error restored (Exp Err).

Comparison of exponential smoothing



If we substitute the original prices, $P_1, \dots, P_5$, into the equation

$$DMA_5 = (P_1 + 2P_2 + 3P_3 + 2P_4 + P_5) / 9$$

This shows that double smoothing results in weighting the end values, rather than the original equal weighting of exponential smoothing the result is different. Because the weight of the smoothing constant, a , the double smoothing

$$DE_t = DE_{t-1} + a(E_t - DE_{t-1})$$

causes the nearby value t to be smoothed twice by a , or a^2 . Therefore, the net effect of using a smoothing of $a = .10$ is much more closely to using the square root of a , approximately .32. Examples 4-4 and 4-5 give an example of this method.

Blau's Double Smoothing

To avoid this compounded lag, William Blau has substituted the price itself.⁴ The smoothing is then performed on the smoothed result restores the speed of the series back to the original. Changes are called the *first difference*, or *speed*. In effect, the smoothing has a lag. Blau has found this to be a successful proxy for the first smoothing may be as long as 300 periods and the second smoothing (also see Table 4-4).

⁴ William Blau, *Momentum, Direction, and Divergence* (John Wiley & Sons, 1980).

Comparing Exponential Smoothing Methods

Figure 44 and Table 44 illustrate the differences between a simple exponential smoothing (Exp) with smoothing constant $\alpha = .10$, smoothing with the lag (or forecast error) restored (Exp Err) using the same $\alpha = .10$, and double smoothing (Dbl Exp) with the same smoothing constant. In the following example, a trend was defined as the direction of the unfiltered trendline. When the change in the trendline value was positive, the trend was up; when it was negative, the trend was down.

The single exponential, Exp, appears as the medium-term trend in the sample period of about 4 1/2 months. It had 9 distinct trend periods and a net profit, without any transaction costs, of 2.59 points. Double smoothing was predictably slower, showing a much smoother curve and only 4 trend changes. It netted only .07 points, but this particular data sample had a change of direction midway, which does not allow for long-term profits. Restoring the error, Exp Err, only increased the number of trends to 12 and gave a profit of .83 points, despite its appearance as a very fast trend.

Blau's double smoothing of the price changes was especially interesting. It performed differently from the other smoothing methods, yet posted 10 trends, making it similar in speed to the simple exponential, rather than the double-smoothed. Profits of 1.67 points were also high for this short sample, and the trend patterns that it identified were significantly different.

These alternatives all offer good possibilities for identifying the trend and should not be judged by the small sample used in the example.

RELATING EXPONENTIAL SMOOTHING AND STANDARD MOVING AVERAGES

It is much easier to visualize the amount of smoothing in a 10-day moving average than exponential smoothing in which $\alpha = .10$ (which can be called 10% smoothing). Although we try to relate the speed of both techniques, the simple moving average is equally weighted, and the exponential is frontweighted; therefore, they will produce a very different pattern. Because of the inclusion of old data, a 50% smoothing appears slower than a 2-day moving average, a 10% smoothing is slower than a 10-day moving average, and a 5% smoothing is slower than a 20-day moving average. The important factor is that for any specified smoothing constant, the exponential moving average includes all prior data. If a 5-day moving average was compared with an exponential with only 5 total days included, the relationship would be closer to a straight moving average than if the exponential had 10 or 20 days of elapsed calculations.

Instead of using prices, a series of numbers from 1 through 15 and back to 1 will be used to compare a 5-day moving average with an exponential moving average. The exponential is calculated two ways: once using only the last five prices (a modified approach for our example); the other using all prices from the beginning (the standard method).

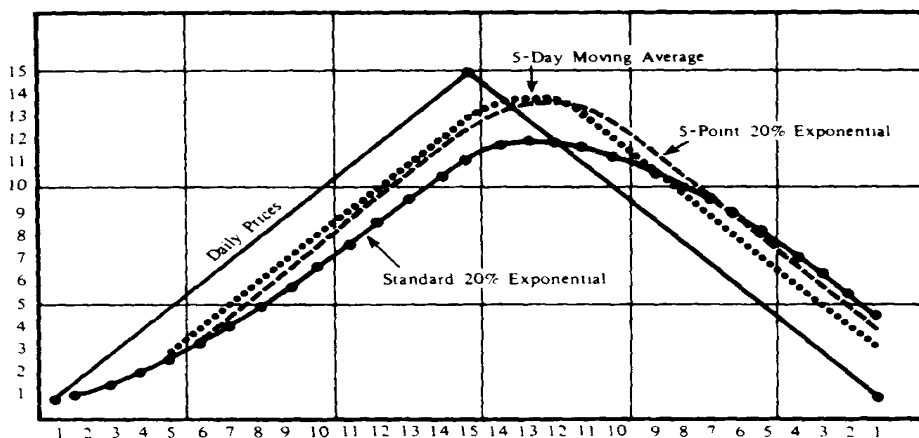
Figures 45 and **46** and Table 45 show the relationship between the standard exponential and the modified exponential using 5 points.

During the period of constant price increase and decrease of at least 5 consecutive days, both the 5day moving average and exponential with five points stabilize; those 5 days represent their entire set of calculation values. At the peak, the standard moving average reacts more quickly than the other methods in staying closer to the current price; the 5point exponential gives 20% of its weight to the most recent price, and less to prior prices, causing it to react more slowly than the standard moving average.

The standard exponential smoothing is different, lagging farther behind each day but increasingly approaching the value of one, as the data increase by one, for long time periods. The weighting of the near values is offset by the retained significance of the oldest data, which is never fully lost, causing the exponentially smoothed moving average to lag,

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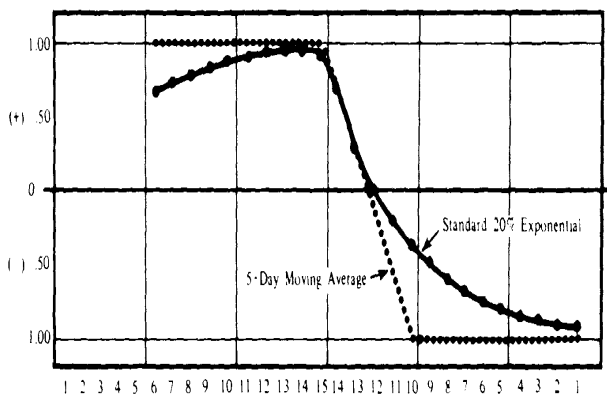
FIGURE 45 Daily price with moving averages.



the farthest behind the current prices. Although there are 14 days of constant decline, the standard exponential has not yet stabilized, still reflecting the turning of prices from up to down at 15.

To form the specific relationship between exponential smoothing and standard moving averages, create a table showing the significance of each oldest day in the exponential calculation. In Table 46, the .50 smoothing constant gives 50% of the total value to the current price, 25% to the prior day, 12.5% to the next oldest, until the 7th oldest day adds only .8% to the total value of the exponential moving average. Table 47 accumulates these weights to show how much of the calculation has been completed by the elapsed days printed across the top. Table 47 is plotted in Figure 47. The most recent days (on the left) receive the bulk of the significance; the oldest prices are of little impact. The .10 smoothing calculation is 90.1% complete by day 22; the total remaining days added together only account for 9.9% of the value.

FIGURE 46 Comparative changes in straight and exponential moving averages.



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TABLE 45 Comparison of Lag between Standard and Exponentially

Price	Standard 5-Day	5-Point 20% Exp	Standard 20% Exp	Change		
				5-Day	5-20% Exp	20% Exp
1	—	1.00	1.00	—	—	—
2	—	1.20	1.20	—	—	—
3	—	1.56	1.56	—	Not enough	—
4	—	2.05	2.05	—	data	—
5	3	2.64	2.64	—	—	—
6	4	3.64	3.31	1.00	1.00	.67
7	5	4.64	4.05	1.00	1.00	.74
8	6	5.64	4.84	1.00	1.00	.79
9	7	6.64	5.67	1.00	1.00	.83
10	8	7.64	6.54	1.00	1.00	.87
11	9	8.64	7.43	1.00	1.00	.89
12	10	9.64	8.34	1.00	1.00	.91
13	11	10.64	9.27	1.00	1.00	.93
14	12	11.64	10.22	1.00	1.00	.95
15	13	12.64	11.17	1.00	1.00	.95
14	13.6	13.24	11.74	.60	.60	.57
13	13.8	13.52	11.99	.20	.28	.25
12	13.6	13.54	11.99	-.20	.02	.00
11	13	13.36	11.79	-.60	-.18	-.20
10	12	12.36	11.43	-1.00	-1.00	-.36
9	11	11.36	10.95	-1.00	-1.00	-.48
8	10	10.36	10.36	-1.00	-1.00	-.59
7	9	9.36	9.69	-1.00	-1.00	-.67
6	8	8.36	8.95	-1.00	-1.00	-.74
5	7	7.36	8.16	-1.00	-1.00	-.79
4	6	6.36	7.33	-1.00	-1.00	-.83
3	5	5.36	6.46	-1.00	-1.00	-.87
2	4	4.36	5.57	-1.00	-1.00	-.89
1	3	3.36	4.65	-1.00	-1.00	-.92

Smoothed Moving Averages

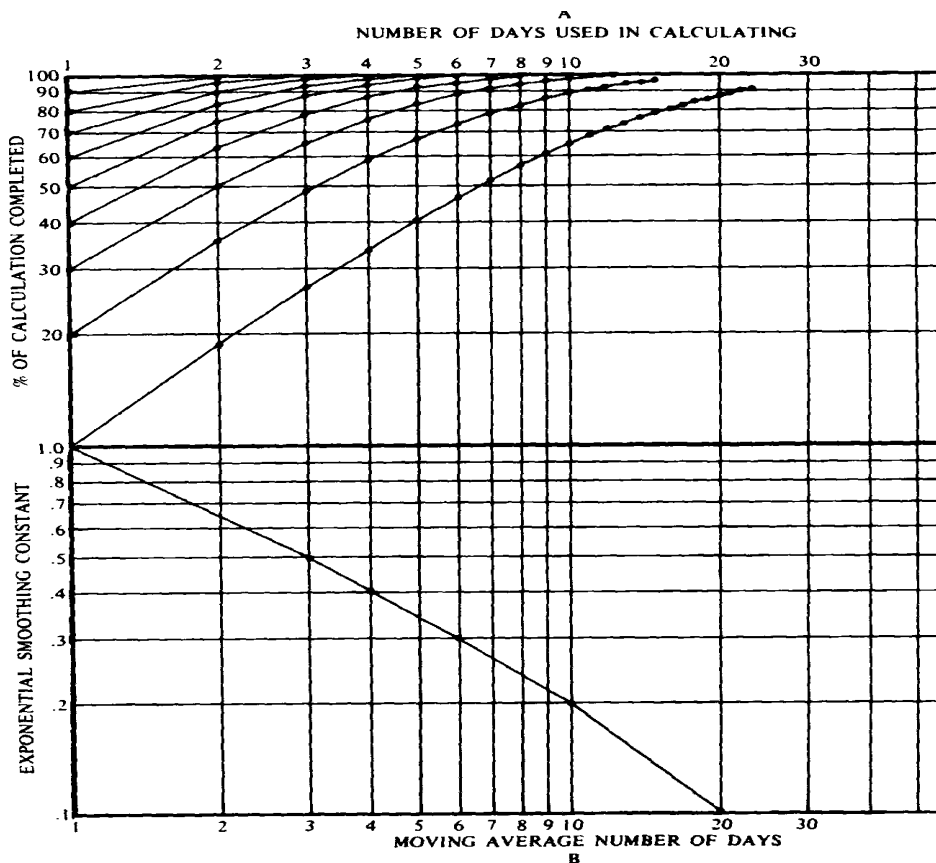
Figure 47 relates the fully calculated exponential smoothing (within 1%) to the standard moving average. Find the smoothing constant on the left and the number of days in a standard moving average along the bottom. Observe that, if you perform an optimization with equally spaced exponential smoothing constants, there is an unexpected distribution relative to past days.

Equating Standard Moving Averages to Exponential Smoothing

TABLE 4-7 Evaluation of Exponential Smoothing—Significance of Prior Data

Weighting (%)	Total Inclusion through Nth Day																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.90	100.0	99.0	99.9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.80	90.0	86.0	99.2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.70	70.0	91.0	97.3	99.2	99.8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.60	60.0	84.0	93.6	97.4	99.0	99.6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.50	50.0	75.0	87.5	93.8	96.9	98.5	99.3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.40	40.0	64.0	78.4	87.0	92.2	95.3	97.2	98.3	99.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.30	30.0	51.0	65.7	76.0	83.2	88.2	91.8	94.2	96.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—
.20	20.0	36.0	48.8	59.0	67.2	73.8	79.0	83.2	86.6	89.3	91.4	93.1	94.5	95.6	96.5	—	—	—	—	—	—	—	—
.10	10.0	19.0	27.1	34.4	41.0	46.8	52.2	57.0	61.3	65.1	68.6	71.8	74.6	77.1	79.4	81.5	83.3	85.0	86.5	87.8	89.1	90.1	91.1

FIGURE 47 Evaluation of exponential smoothing.



number of days. This process will be important when finding robust system parameters and will be discussed in Chapter 21 ("Testing and Optimization").

Equating Exponential Smoothing to Standard Moving Averages

Standard (n-day average)	2	4	6	8	10	12
Smoothing constant (%)	.65	.40	.30	.235	.20	.165

The distribution of smoothing constants is very close to logarithmic and is plotted on a log scale in Figure 46. To test exponential smoothing within a range of days, use a log. arithmic distribution of smoothing constants across that range, with closer values taken at the smaller numbers, if necessary. This may seem an unnecessary precaution, but it is not.

TABLE 4-8 Comparison of Exponential Smoothing Values

Moving Average Days (<i>n</i>)	1st-Order <i>p</i> = 1	2nd-Order <i>p</i> = 2	3rd-Order <i>p</i> = 3
3	.500	.293	.206
5	.333	.184	.126
7	.250	.134	.091
9	.200	.106	.072
11	.167	.087	.059
13	.143	.074	.050
15	.125	.065	.044
17	.111	.057	.039
19	.100	.051	.035
21	.091	.047	.031

Best Approximation of Smoothing Constants

The standard conversion from the number of days to a smoothing constants was given by

Hutson⁵ as

$$s = \frac{2}{n + 1}$$

where *n* is the equivalent number of days in the standard (lag). In addition, 2nd- or 3rd-order exponential smoothing, past 2 or 3 days' prices, may be desirable. This is the exponential smoothing. Its general form is

$$s_p = 1 - \left(1 - \frac{2}{n + 1} \right)^{(1/p)}$$

A comparison of the standard moving average days with 1st-order exponential smoothing is shown in Table 4-8.

Estimating Residual Impact

The primary difference between the standard moving average and exponential smoothing is that prices impact the exponentially smoothed value indefinitely. For practical purposes, the effect of the oldest data may be limited. A general method of approximating the smoothing constant *s* for a given level of residual impact is given by

$$s = 1 - RI^{1/n}$$

⁵ Jack K. Hutson, "Filter Price Data: Moving Averages vs. *Stocks & Commodities* (May/June 1984).

TABLE 4-9 Comparison of Exponential Smoothing Residual Impact

<i>n</i>	$2/(n + 1)$	<i>RI</i> (%)	10% <i>RI</i>	5% <i>RI</i>
5	.333	13.17	.369	.451
10	.182	13.44	.206	.259
15	.125	13.49	.142	.181
20	.095	13.51	.109	.139

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where *RI* is the level of residual impact (e.g., .05, .10, .20). A lower *RI* means more residual impact, and *n* is the equivalent number of moving averages.

The approximation for the smoothing constant, given in the preceding formula (1), can be shown to have a consistent residual impact of between 13 and 15 percent. The preceding formula, as shown in Table 4-9, would allow the specific residual impact.

⁵ Donald R. Lambert, "Exponentially Smoothed Moving Averages," *Technical Analysis of Stocks & Commodities* (September/October 1984).